Reg. No. :

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B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

Fourth Semester

Computer Science and Engineering

MA 6453 — PROBABILITY AND QUEUEING THEORY

(Common to Mechanical Engineering (Sandwich) and Information Technology)

(Regulations 2013)

Time : Three hours | Maximum : 100 marks

Use of statistical tables may be permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Test whether $f(x) = \begin{cases} |x|, & -1 \le x \le 1 \\ 0, & otherwise \end{cases}$ can be the probability density function of a continuous random variable?

2. What are the limitations of Poisson distribution?

3. Given the two regression lines $3X + 12Y = 19$, $3Y + 9X = 46$, find the coefficient of correlation between X and Y.

4. The joint probability density function of bivariate random variable (X, Y) is given by $f(x, y) = \begin{cases} 4xy, 0 < x < 1, \ 0 < y < 1 \\ 0 \qquad elsewhere \end{cases}$. find $P(X + Y < 1)$.

5. Define a k^{th} order stationary process. When will it become a strict sense stationary process?

6. State Chapman Kolmogorov theorem.

7. For an $M/M/C/N$ FCFS (C<N) queueing system, write the expressions for P_0 and P_N.

8. Define

   (a) balking and

   (b) reneging of the customers in the queueing system.

9. State the formula for the probability that there are n customers in the system of $(M/M/1):(FIFO/N/\infty)$.

10. Define: Open Jackson networks.

PART B — (5 × 16 = 80 marks)

11. (a) (i) The density function of a continuous random variable X is given by

$$f(x)=\begin{cases} ax, & 0\le x\le 1 \\ a, & 1\le x\le 2 \\ 3(a-x), & 2\le x\le 3 \\ 0 & otherwise \end{cases}$$

(1) Find the value of 'a' (2) CDF of X (8)

(ii) The probability of a man hitting a target is 1/4. If he fires 7 times, what is the probability of his hitting the target at least twice? And how many times must he fire so that the probability of his hitting the target at least once is greater than 2/3? (8)

Or

(b) (i) Find the MGF of the random variable X having the probability density function

$$f(x)=\begin{cases} \frac{x}{4}e^{-x/2} & x>0 \\ 0 & otherwise \end{cases}$$

. Also find the first four moments about the origin. (8)

(ii) In a normal population with mean 15 and standard deviation 3.5, it is found that 647 observations exceed 16.25. What is the total number of observations in the population? (8)

12. (a) (i) Find the equation of the regression line Y on X from the following data : (10)

X	3	5	6	8	9	11
Y	2	3	4	6	5	8

(ii) Assume that the random variables X and Y have the joint PDF $f(x,y) = \frac{1}{2}x^3y; 0 \le x \le 2, 0 \le y \le 1$. Determine if X and Y are independent. (6)

Or

(b) The joint PDF of the random variables X and Y is defined as

$$f(x,y) = \begin{cases} 25e^{-5y}; 0 < x < 0.2, y > 0 \\ 0, \ otherwise \end{cases}$$

(i) Find the marginal PDFs and X and Y

(ii) What is the covariance of X and Y? (16)

13. (a) (i) Show that the random process $X(t) = A\cos wt + B\sin wt$ is wide sense stationary process if A and B are random variables such that $E(A) = E(B) = 0$, $E(A^2) = E(B^2)$ and $E(AB) = 0$. (6)

(ii) A machine goes out of order whenever a component part fails. The failure of this part is in accordance with a Poisson process with a mean rate of 1 per week. Find the probability that 2 weeks have lapsed since the last failure. If there are 5 spare parts of this component in an inventory and the next supply is not due in 10 weeks, find the probability that the machine will not be out of order in the next 10 weeks. (10)

Or

(b) Consider a Markov chain on (0,1,2) having the transition matrix given by $P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ Show that the chain is irreducible. Find the period and the stationary distribution. (16)

14. (a) (i) Customers arrive at a watch repair shop according to a Poisson process at a rate of 1 per every 10 minutes, and the service time is an exponential random variable with mean 8 minutes. Compute

(1) the mean number of customers L_s in the system. (2)

(2) the mean waiting time W_s of a customer spends in the system, (2)

(3) the mean waiting W_q of a customer spends in the queue, (2)

(4) the probability that the server is idle. (2)

(ii) A petrol pump station has 4 petrol pumps. The service time follows an exponential distribution with mean of 6 minutes and cars arrive for service in a Poisson process at the rate of 30 cars per hour. (8)

(1) Find the probability that no car is in the system.

(2) What is the probability that an arrival will have to wait in the queue?

(3) Find the mean waiting time in the system.

Or

(b) (i) A one person barber shop has 6 chairs to accommodate people waiting for a haircut. Assume that customers who arrive when all the 6 chairs are full leave without entering the barber shop. Customers arrive at the rate of 3 per hour and spend an average of 15 minutes in the barber's chair. Compute

(1) P_0

(2) L_q

(3) P_7

(4) W_s (8)

(ii) Consider a single-server queue where the arrivals are Poisson with rate $\lambda = 10$/hour The service distribution is exponential with rate $\mu = 5$/hour. Suppose that customers balk at joining the queue when it is too long. Specifically, when there are 'n' in the system, an arriving customer joins the queue with probability $\frac{1}{(n+1)}$ Determine the steady-state probability that there are $'n'$ customers in the system. (8)

15. (a) Derive the Pollaczek - Khinchjn formula for M/G/1 queue. Hence deduce the result for the queues M/D/l and M/Ek/1 as special cases. (16)

Or

(b) Consider a system two servers where customers arrive from outside the system in a Poisson fashion at server 1 at a rate of 4 / hour and at server 2 at a rate of 5/hour. The customers are served at station 1 and station 2 at the rate of 8 / hour and 10 / hour respectively. A customer, after completion of service at server 1 is equally likely to go to server 2 or to leave the system. A departing Customer from server 2 will go to server 1 twenty five percent of the time and will depart from the system otherwise Find the total arrival rates at server 1 and server 2. Find the limiting probability of n customers at server 1 and m customers at server 2. Find the expected number of customers in the system.